Sample Question Paper - 3
Mathematics (041)
Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours **Maximum Marks: 40**

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate: $\int \frac{\cos x}{\sin^2 x+4\sin x+5}dx$ **[2]**

OR

If $\int_0^a \sqrt{x}dx = 2a\int_0^{\pi/2} \sin^3 x\, dx$, find the value of integral $\int_a^{a+1} x dx$

2. Find the general solution of $\left(1+x^2\right) dy + 2xydx = \cot x dx (x \neq 0)$ **[2]**
3. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular unit vectors, then prove that $|\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$ **[2]**
4. Find the distance between the point P(6, 5, 9) and the plane determined by the points A (3, -1, 2), B (5, 2, 4) and C(-1, -1, 6). **[2]**
5. The probability that a bulb produced by a factory will fuse after 6 months of use is 0.05. Find the probability that out of 5 such bulbs at least one will fuse after 6 months of use. **[2]**
6. In a school there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl? **[2]**

Section B

7. Evaluate: $\int \frac{(3x+5)}{(x^3-x^2+x-1)}dx$. **[3]**
8. It is given that the rate at which some bacteria multiply is proportional to the instantaneous number present. If the original number of bacteria doubles in two hours, in how many hours will it be five times? **[3]**

OR

Solve the differential equation: $\frac{dy}{dx} - y = xe^x$

9. If $\vec{a}, \vec{b}, \vec{c}$ are vectors such that $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c}, \vec{a}\times\vec{b} = \vec{a}\times\vec{c}, \vec{a} \neq \overrightarrow{0}$, then show that $\vec{b} = \vec{c}$. **[3]**
10. Find the equation of line passing through points A (0,6,-9) and B(-3,-6,3). If D is the foot of perpendicular drawn from the point C (7,4,-1) on the line AB, then find the coordinates of point D and equation of line CD. **[3]**

OR

Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i} + 5\hat{j} - 6\hat{k}$

Section C

11. Evaluate the integral: $\int \frac{|\cot x+\cot^3 x|}{1+\cot^3 x} dx$ **[4]**

12. Find the area bounded by the circle $x^2 + y^2 = 16$ and the line $\sqrt{3}y = x$ in the first quadrant, using integration. **[4]**

OR

Find the area of the region in the first quadrant enclosed by *x*-axis, line $x = \sqrt{3}y$ and the given curve $x^2 + y^2 = 4$

13. Find the equation of the plane through the points (2, 1, -1) and (-1, 3, 4), and perpendicular to the plane x - 2y + 4z = 10. **[4]**

CASE-BASED/DATA-BASED

14. Elpis Limited is a company that produces electric bulbs. The quality of their bulbs is really very good. The customers are well satisfied and it has been as well recommended brand in the market. The probability that a bulb produced by Elpis Limited will fuse after 150 days of use is 0.05. **[4]**

Find the probability that out of 5 such bulbs

i. No bulb will fuse after 150 days of use.
ii. Not more than one will fuse after 150 days of use.

Solution

MATHEMATICS 041

Class 12 - Mathematics

Section A

1. Let I = $\int \frac{\cos x}{\sin^2 x+4\sin x+5}dx$

   Also let sin x = t then cos x dx = dt

   So, $I=\int \frac{dx}{t^2+4t+5}$

   $=\int \frac{dt}{t^2+2t(2)+(2)^2-(2)^2+5}$

   $=\int \frac{dt}{(t+2)^2+1}$

   Again, Let (t + 2) = u then dt = du

   $I=\int \frac{dt}{u^2+1}$

   = $\tan^{-1}$ (u) + c [Since, $\int \frac{dt}{u^2+1}$ dx = $\tan^{-1}$ x + c]

   I = $\tan^{-1}$ (t + 2) + c

   I = $\tan^{-1}$ (sin x + 2) + c

   OR

   We have,

   $\int_0^a \sqrt{x}dx=\frac{2}{3}\left[x^{3/2}\right]_0^a=\frac{2}{3}a^{3/2}$ ....(i)

   Let $I=\int_0^{\pi/2}\sin^3 xdx$, then

   $I=\int_0^{\pi/2}\frac{3\sin x-\sin 3x}{4}dx=\frac{1}{4}\int_0^{\pi/2}(3\sin x-\sin 3x)dx=\frac{1}{4}\left[-3\cos x+\frac{1}{3}\cos 3x\right]_0^{\pi/2}$

   $\Rightarrow \quad I=\frac{1}{4}\left[\left(-3\cos\frac{\pi}{2}+\frac{1}{3}\cos\frac{3\pi}{2}\right)-\left(-3+\frac{1}{3}\right)\right]$

   $=\frac{1}{4}\left[0-\left(-3+\frac{1}{3}\right)\right]=\frac{1}{4}\left[3-\frac{1}{3}\right]=\frac{2}{3}\ .....(ii)$

   It is given that $\int_0^a \sqrt{x}dx=2a\int_0^{\pi/2}\sin^3 xdx$

   $\Rightarrow \frac{2}{3}a^{3/2}=2a\left(\frac{2}{3}\right)$

   $\Rightarrow a^{3/2}=2a \Rightarrow a^3=4a^2 \Rightarrow a^2(a-4)=0$

   $\Rightarrow a=0,4$ [Using(i) and (ii)]

   When a = 4, we get

   $\int_a^{a+1} xdx=\int_4^5 xdx=\left[\frac{x^2}{2}\right]_4^5=\frac{25}{2}-\frac{16}{2}=\frac{9}{2}$

   When a = 0, we get

   $\int_a^{a+1} xdx=\int_0^1 xdx=\left[\frac{x^2}{2}\right]_0^1=\frac{1}{2}-0=\frac{1}{2}$

   Hence, $\int_a^{a+1} xdx=\frac{9}{2}\ or, \frac{1}{2}$

2. It is given that $\left(1+x^2\right)dy+2xydx=cotxdx$

   $\Rightarrow \frac{dy}{dx}+\frac{2xy}{(1+x^2)}=\frac{\cot x}{1+x^2}$

   This is equation in the form of $\frac{dy}{dx}+py=Q$ (where $p=\frac{2x}{(1+x^2)}$ and $Q=\frac{\cot x}{1+x^2}$)

   Now, I.F. = $e^{\int pdx}=e^{\int \frac{2x}{(1+x^2)}dx}=e^{\log(1+x^2)}=1+x^2$

   Thus, the solution of the given differential equation is given by the relation:

   $\mathrm{y(I.F)=\int(Q\times I.F)dx+C}$

   $\Rightarrow \mathrm{y}\cdot\left(1+\mathrm{x}^2\right)=\int\left[\frac{\cot x}{1+\mathrm{x}^2}\cdot\left(1+\mathrm{x}^2\right)\right]\mathrm{dx}+\mathrm{C}$

   $\Rightarrow \mathrm{y}\cdot\left(1+\mathrm{x}^2\right)=\int \cot \mathrm{xdx}+\mathrm{C}$

   $\Rightarrow \mathrm{y}\left(1+\mathrm{x}^2\right)=\log|\sin \mathrm{x}|+\mathrm{C}$

   Therefore, the required general solution of the given differential equation is

   $y\left(1+x^2\right)=\log|\sin x|+C$

3. Given that a, b, c are mutually prependicular vectors,

So,
$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$....(1)
Also, a,b and c are unit vectors, so
$|\vec{a}|=|\vec{b}|=|\vec{c}|=1$
$|\vec{a}+\vec{b}+\vec{c}|^2=(\vec{a}+\vec{b}+\vec{c})^2$
$=(\vec{a})^2+(\vec{b})^2+(\vec{c})^2+2\vec{a}.\vec{b}+2\vec{b}.\vec{c}+2\vec{c}.\vec{a}$
$=|\vec{a}|^2+|\vec{b}|^2+|\vec{c}|^2+2(0)+2(0)+2(0)$[from (1)]
$=(1)^2+(1)^2+(1)^2+0$
$|\vec{a}+\vec{b}+\vec{c}|^2=1+1+1$
$|\vec{a}+\vec{b}+\vec{c}|^2=3$
$|\vec{a}+\vec{b}+\vec{c}|=\sqrt{3}$

4. Let A, B, C be the three points in the plane. D is the foot of the perpendicular drawn from a point P to the plane. PD is the required distance to be determined, which is the projection of $\overrightarrow{AP}$ on $\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}$
Hence, PD = the dot product of $\overrightarrow{AP}$ with the unit along $\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}$
So, $\overrightarrow{\mathrm{AP}}=3\hat{i}+6\hat{j}+7\hat{k}$
and $\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&2\\-4&0&4\end{vmatrix}=12\hat{i}-16\hat{j}+12\hat{k}$
Unit vector along $\overrightarrow{\mathrm{AB}}\times\overrightarrow{\mathrm{AC}}=\frac{3\hat{i}-4\hat{j}+3\hat{k}}{\sqrt{34}}$
Hence $\mathrm{PD}=(3\hat{i}+6\hat{j}+7\hat{k})\cdot\frac{3\hat{i}-4\hat{j}+3\hat{k}}{\sqrt{34}}$
= $\frac{3\sqrt{34}}{17}$

5. Let X represent the number of bulbs that will fuse after 6 months of use in an experiment of 5 trials. The trials are Bernoulli trials.
It is given that, p = 0.05
∴ q = 1 −p = 1 − 0.05 = 0.95
X has a binomial distribution with n = 5 and p = 0.05
∴ $P(X = x) = {}^{n}C_{x}q^{n-x}p^{x}$, where x = 1,2,...n
$= {}^{5}C_{x}(0.95)^{5-x}(0.05)^{x}$
P(at least one) = P(X ≥ 1)
1-P(X < 1)
=1 − P(X = 0)
$=1-{}^{5}C_{0}(0.95)^{5}\times(0.05)^{0}$
$=1-1\times(0.95)^{5}$
$=1-(0.95)^{5}$

6. Let E denotes the event that student chosen randomly studies in class XII, F denotes the event that randomly chosen student is girl.
P (E|F) = ?
$P(F)=\frac{430}{1000}=0.43$
$P(E\cap F)=\frac{43}{1000}=0.043$
$P\left(\frac{E}{F}\right)=\frac{P(E\cap F)}{P(F)}$
$=\frac{0.043}{0.43}=0.1$

Section B

7. Let the given integral be, $I=\int\frac{3x+5}{(x^2-x^2+x-1)}dx$
Now by partial fractions putting, $\frac{3x+5}{(x^3-x^2+x-1)}=\frac{A}{x-1}+\frac{Bx+C}{(x^2+1)}$

$A(x^2 + 1) + (Bx + C)(x - 1) = 3x + 5$
Putting x - 1 = 0,
X = 1
A(2) + B(0) = 3 + 5 = 8
A = 4
By equating the coefficient of x^2 and constant term, A + B = 0
4 + B = 0
B = -4
A - C = 5
4 - C = 5
C = -1
From equation (1),we get,
$\frac{3x+5}{(x-1)(x^2+1)} = \frac{4}{x-1} + \frac{-4x-1}{(x^2+1)}$
$\int \frac{3x+5}{(x-1)(x^2+1)} dx = 4\int \frac{1}{x-1} dx - 4\int \frac{1}{(x^2+1)} dx - \int \frac{1}{(x^2+1)} dx$
$= 4\log(x-1) - \frac{4}{2}\log\left(x^2+1\right) - \tan^{-1} x + c$
$= 4\log(x - 1) - 2\log(x^2 + 1) - \tan^{-1}x + c$

8. Let the original count of bacteria be N_0 and at any time t the count of bacteria be N. We have,
$\frac{dN}{dt} \propto N$
$\Rightarrow \frac{dN}{dt} = \lambda N$, where λ is a constant
$\Rightarrow \frac{dN}{N} = \lambda dt$
$\Rightarrow \int \frac{1}{N} dN = \lambda \int dt$ .....(i)
$\Rightarrow \log N = \lambda t + C$
We have, $N = N_0$ at t = 0. Putting t = 0 and $N = N_0$ in (i), we get,
$\therefore q \log N_0 = 0 + C \Rightarrow C = \log N_0$
Putting $C = \log N_0$ in (i), we have,
$\log N = \lambda t + \log N_0$
$\Rightarrow \log\left(\frac{N}{N_0}\right) = \lambda t$ ...(ii)
It is given that the original number of bacteria doubles in 2 hrs.
That is when t = 2 hours, $N = 2 N_0$. Put t = 2 and $N = 2N_0$ in (ii), we have,
$\log\left(\frac{2N_0}{N_0}\right) = 2\lambda \Rightarrow \lambda = \frac{1}{2}\log 2$
Putting $\lambda = \frac{1}{2}\log 2$ in (ii), we have,
$\log\left(\frac{N}{N_0}\right) = \left(\frac{1}{2}\log 2\right) t$
$\Rightarrow t = \frac{2}{\log 2}\log\left(\frac{N}{N_0}\right)$ ...(iii)
Suppose the count of bacteria becomes 5 times i.e. $5 N_0$ in t_1 hours. Putting $t = t_1$ and $N = 5N_0$ (iii), we have,
$t_1 = \frac{2}{\log 2}\log\left(\frac{5N_0}{N_0}\right) = \frac{2}{\log 2}(\log 5) = \frac{2\log 5}{\log 2}$ hours.

OR

The given differential equation is,
$\frac{dy}{dx} - y = xe^x$
It is a linear differential equation. Comparing it with,
$\frac{dy}{dx} + Py = Q$
$P = -1, Q = xe^x$
$\text{I.F.} = e^{\int p dx}$
$= e^{-\int dx}$
$= e^{-x}$
Solution of the equation is given by,
$y \times (\text{I.F.}) = \int Q \times (\text{I.F.})\, dx + c$

$ye^{-x} = \int xe^x \times e^{-x}\, dx + c$

$= \int x\, dx + c$

$ye^{-x} = \frac{x^2}{2} + c$

$y = e^x \left(\frac{x^2}{2} + c\right)$

9. Given ,

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} = 0$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \vec{a} \cdot (\vec{b} - \vec{c}) = 0$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \vec{b} - \vec{c} = \vec{0}$ or $\vec{a} \perp (\vec{b} - \vec{c})$ $[\because \vec{a} \neq \vec{0}]$

$\Rightarrow \vec{b} = \vec{c}$ or, $\vec{a} \perp (\vec{b} - \vec{c})$ ...(i)

Again given,

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = \vec{0}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$ and $\vec{a} \neq \vec{0}$

$\Rightarrow \vec{b} - \vec{c} = \vec{0}$ or $\vec{a} \| (\vec{b} - \vec{c})$ $[\because \vec{a} \neq \vec{0}]$

$\Rightarrow \vec{b} = \vec{c}$ or $\vec{a} \| (\vec{b} - \vec{c})$ ...(ii)

From (i) and (ii), it follows that $\vec{b} = \vec{c}$, because $\vec{a}$ cannot be both parallel and perpendicular to vectors $(\vec{b} - \vec{c})$

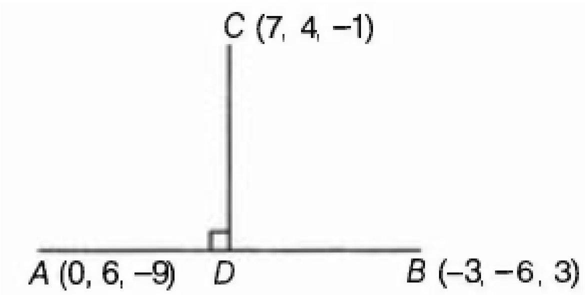
10. We have to find the equation of line passing through points A (0,6,-9) and B(-3,-6,3). If D is the foot of perpendicular drawn from the point C (7,4,-1) on the line AB, then we have to find the coordinates of point D and equation of line CD.

We know that, equation of line passing through the points $(x_1, y_1, z_1)$and $(x_2, y_2, z_2)$is given by

$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$.......(i)

Here, $A(x_1, y_1, z_1) = (0,6,-9)$

and $B(x_2, y_2, z_2) = (-3,-6,3)$



$\therefore$ Equation of line AB is given by,

$\frac{x - 0}{-3 - 0} = \frac{y - 6}{-6 - 6} = \frac{z + 9}{3 + 9}$

$\Rightarrow \quad \frac{x}{-3} = \frac{y - 6}{-12} = \frac{z + 9}{12}$

$\Rightarrow \quad \frac{x}{-1} = \frac{y - 6}{-4} = \frac{z + 9}{4}$ [dividing denominator by 3]

Next, we have to find coordinates of foot of perpendicular D.

Now, let $\frac{x}{-1} = \frac{y - 6}{-4} = \frac{z + 9}{4} = \lambda$(say)

$\Rightarrow \quad x = -\lambda,$

$y - 6 = -4\lambda$ and $z + 9 = 4\lambda$

$\Rightarrow \quad x = -\lambda, y = -4\lambda + 6$ and $z = 4\lambda - 9$

Since CD lies on line AB ,so coordinates of ,

$D = (-\lambda, -4\lambda + 6, 4\lambda - 9)$....(ii)

Now, DR's of line CD are

$(-\lambda - 7, -4\lambda + 6 - 4, 4\lambda - 9 + 1)$

$= (-\lambda - 7, -4\lambda + 2, 4\lambda - 8)$

Now, CD⊥AB

∴ $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Where, $a_1 = -\lambda - 7, b_1 = -4\lambda + 2, c_1 = 4\lambda - 8$ [DR's of line CD]

and $a_2 = -1, b_2 = -4, c_2 = 4$ [DR's of line AB]

$\Rightarrow (-\lambda - 7)(-1) + (-4\lambda + 2)(-4) + (4\lambda - 8)4 = 0$

$\Rightarrow \quad \lambda + 7 + 16\lambda - 8 + 16\lambda - 32 = 0$

$\Rightarrow \quad 33\lambda - 33 = 0$

$\lambda = 1$

On putting $\lambda = 1$ in Eq. (ii), we get required foot of perpendicular,

D = (-1,2,5)

Also, we have to find equation of line CD, where

C(7,4,-1) and D(-1,2,-5).

∴ Required equation of line is

$\frac{x-7}{-1-7} = \frac{y-4}{2-4} = \frac{z+1}{-5+1}$ [using Eq. (i)]

$\Rightarrow \quad \frac{x-7}{-8} = \frac{y-4}{-2} = \frac{z+1}{-4}$

$\Rightarrow \quad \frac{x-7}{4} = \frac{y-4}{1} = \frac{z+1}{2}$ [dividing denominator by -2]

OR

$\vec{n} = 3\hat{i} + 5\hat{j} - 6\hat{k}$

$|(\vec{n})| = \sqrt{9 + 16 + 144} = \sqrt{70}$

$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$

$= \frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}$

$\vec{r}.\hat{n} = 7$

$r.\left(\frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}\right) = 7$

Section C

11. Let the given integral be,

$I = \int\left(\frac{\cot x + \cot^3 x}{1 + \cot^3 x}\right) dx$

$= \int\left[\frac{\cot x\left(1 + \cot^2 x\right)}{1 + \cot^3 x}\right] dx$

$= \int\left(\frac{\cot x \csc^2 x}{1 + \cot^3 x}\right) dx$

Putting cot x = t

$\Rightarrow -\text{cosec}^2 x\, dx = dt$

$\Rightarrow \text{cosec}^2 x\, dx = -dt$

$\therefore I = -\int \frac{tdt}{1 + t^3}$

$= -\int \frac{tdt}{(1+t)\left(t^2 - t + 1\right)}$

Using partial fraction let $\frac{t}{(1+t)\left(t^2-t+1\right)} = \frac{A}{t+1} + \frac{Bt+C}{t^2-t+1}$

$\Rightarrow \frac{t}{(1+t)\left(t^2-t+1\right)} = \frac{A\left(t^2-t+1\right) + (Bt+C)(t+1)}{(t+1)\left(t^2-t+1\right)}$

$\Rightarrow t = A(t^2 - t + 1) + Bt^2 + Bt + Ct + C$

$\Rightarrow t = (A + B)t^2 + (B + C - A)t + A + C$

Equating Coefficients of like terms

A + B = 0 ...(i)

B + C - A = 1 ...(ii)

A + C = 0 ...(iii)

Solving (i), (ii) and (iii), we get

$A = -\frac{1}{3}$

$B = \frac{1}{3}$

$C = \frac{1}{3}$

$\therefore \frac{t}{(1+t)(t^2-t+1)} = -\frac{1}{3(t+1)} + \frac{1}{3}\left(\frac{t+1}{t^2-t+1}\right)$

$\Rightarrow \frac{t}{(1+t)(t^2-t+1)} = -\frac{1}{3(t+1)} + \frac{1}{6}\left[\frac{2t+2}{t^2-t+1}\right]$

$\Rightarrow \frac{t}{(1+t)(t^2-t+1)} = -\frac{1}{3(t+1)} + \frac{1}{6}\left[\frac{2t-1+3}{t^2-t+1}\right]$

$\therefore I = -\left[-\frac{1}{3}\int \frac{dt}{t+1} + \frac{1}{6}\int\left(\frac{2t-1}{t^2-t+1}\right)dt + \frac{1}{2}\int\frac{dt}{t^2-t+1}\right]$

$= +\frac{1}{3}\int\frac{dt}{t+1} - \frac{1}{6}\int\left(\frac{2t-1}{t^2-t+1}\right)dt - \frac{1}{2}\int\frac{dt}{t^2-t+\frac{1}{4}-\frac{1}{4}+1}$

$= \frac{1}{3}\int\frac{dt}{t+1} - \frac{1}{6}\int\frac{(2t-1)dt}{(t^2-t+1)} - \frac{1}{2}\int\frac{dt}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

let $t^2 - t + 1 = p$

$\Rightarrow (2t - 1)dt = dp$

$\therefore I = \frac{1}{3}\int\frac{dt}{t+1} - \frac{1}{6}\int\frac{dp}{p} - \frac{1}{2}\int\frac{dt}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1}{3}\log|t+1| - \frac{1}{6}\log|p| - \frac{1}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$

$= \frac{1}{3}\log|t+1| - \frac{1}{6}\log|p| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right) + C$

$= \frac{1}{3}\log|\cot x+1| - \frac{1}{6}\log\left|\cot^2 x - \cot x + 1\right| - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\cot x-1}{\sqrt{3}}\right) + C$

12. According to the question ,

Given equation of circle is $x^2 + y^2 = 16$ ...(i)

Equation of line given is ,

$\sqrt{3}y = x$ ...(ii)

$\Rightarrow y = \frac{1}{\sqrt{3}}x$ represents a line passing through the origin.

To find the point of intersection of circle and line ,

substitute eq. (ii) in eq.(i) , we get

$x^2 + \frac{x^2}{3} = 16$

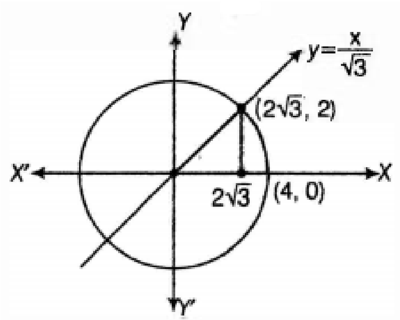
$\frac{3x^2+x^2}{3} = 16$

$\Rightarrow 4x^2 = 48$

$\Rightarrow x^2 = 12$

$\Rightarrow x = \pm 2\sqrt{3}$

When $x = 2\sqrt{3}$, then $y = \frac{2\sqrt{3}}{\sqrt{3}} = 2$



Required area (In first quadrant) = (Area under the line $y = \frac{1}{\sqrt{3}}x$ from x = 0 to $2\sqrt{3}$) + (Area under the circle from x = $2\sqrt{3}$ to x=4)

$= \int_0^{2\sqrt{3}} \frac{1}{\sqrt{3}}x\,dx + \int_{2\sqrt{3}}^{4}\sqrt{16-x^2}\,dx$

$= \frac{1}{\sqrt{3}}\left[\frac{x^2}{2}\right]_0^{2\sqrt{3}} + \left[\frac{x}{2}\sqrt{16-x^2} + \frac{(4)^2}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_{2\sqrt{3}}^{4}$

$= \frac{1}{2\sqrt{3}}\left[(2\sqrt{3})^2 - 0\right] + \left[0 + 8\sin^{-1}(1) - \frac{2\sqrt{3}}{2}\sqrt{16-12} - 8\sin^{-1}\left(\frac{2\sqrt{3}}{4}\right)\right]$

$= 2\sqrt{3} + 8\left(\frac{\pi}{2}\right) - \frac{2\sqrt{3}}{2} \times 2 - 8\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$= 2\sqrt{3} + 4\pi - 2\sqrt{3} - 8\left(\frac{\pi}{3}\right)$

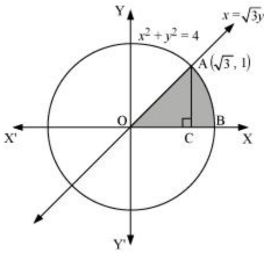
$= 4\pi - \frac{8\pi}{3}$

$= \frac{12\pi - 8\pi}{3}$

$= \frac{4\pi}{3}$ sq units.

OR

The graphical representation of given line and curve



Area OAB = Area ΔOCA + Area ACB

Area of $\triangle OCA = \frac{1}{2} \times base \times height$

$= \frac{1}{2} \times \sqrt{3} \times 1$

$= \frac{\sqrt{3}}{2}$ sq. units ...(1)

Area of $ACB = \int_{\sqrt{3}}^{2} \sqrt{4 - x^2}\, dx$

$= \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_{\sqrt{3}}^{2}$

$= \left[0 + 2 \times sin^{-1}(1) - \frac{\sqrt{3}}{2}\sqrt{4-3} - 2\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$= \left[\pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3}\right]$

$= \left[\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right]$ ...(2)

Area required = area of OAC + Area of ACB

$= \frac{\sqrt{3}}{2} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} = \frac{\pi}{3}$ sq. units

13. The equation of the plane passing through (2, 1, -1) is

a(x - 2) + b(y - 1) + c(z + 1) = 0 ...... (i)

Since, this passes through (-1, 3, 4)

$\therefore a(-1-2) + b(3-1) + c(4+1) = 0$

$\Rightarrow -3a + 2b + 5c = 0$ ..... (ii)

Since, the plane (i) is perpendicular to the plane x - 2y + 4z = 10.

$\therefore 1 \cdot a - 2 \cdot b + 4 \cdot c = 0$

$\Rightarrow a - 2b + 4c = 0$ .... (iii)

On solving Eqs. (ii) and (iii), we get

$\frac{a}{8+10} = \frac{-b}{-17} = \frac{c}{4} = \lambda$

$\Rightarrow a = 18\lambda, b = 17\lambda, \lambda = 4\lambda$

From Eq. (i),

$18\lambda(x-2) + 17\lambda(y-1) + 4\lambda(z+1) = 0$

$\Rightarrow 18x - 36 + 17y - 17 + 4z + 4 = 0$

$\Rightarrow 18x + 17y + 4z - 49 = 0$

∴ 18x + 17y + 4z = 49

CASE-BASED/DATA-BASED

14. Let p = Probability of a success and q = Probability of a failure

p = P (a bulb will fuse after 150 days) = 0.05 and q = 1 – 0.05 = 0.95

n = 5 and P (X = r) = C (n, r) $p^r q^{n-r}$

i. No bulb is fused, r = 0

P (X = 0) = C (5, 0), $(0.05)^0(0.95)^5 = \left(\frac{19}{20}\right)^5 = (0.95)^5$

ii. Not more than one fused bulb

P (not more than one fused bulb) = P (X = 0) + P (X = 1)

$= \left(\frac{19}{20}\right)^5 + C(5,1)\ (0.05)\ (0.95)^4$

$= \left(\frac{19}{20}\right)^5 + 5(0.05)\left(\frac{19}{20}\right)^4$

$= \left(\frac{19}{20}\right)^4 \left(\frac{19}{20} + \frac{5}{20}\right)$

$= \left(\frac{19}{20}\right)^4 \left(\frac{6}{5}\right) = 1.2(0.95)^4$